Midpoint, Distance, Slope, Partitioning

Example: Given one endpoint and a midpoint of a segment, find the other endpoint.

Midpoint: $(-4, 6)$ Endpoint: $(?, ?)$

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

E_1:	2	1
E_2	x	y
M:	-4	6

Solve x

$$\frac{2+x}{2} = -4 \cdot 2$$

$$2 + x = -8$$

$$-2 \quad -2$$

$$x = -10$$

Solve y

$$\frac{1+y}{2} = 6 \cdot 2$$

$$1 + y = 12$$

$$-1 \quad -1$$

$$y = 11$$

$$-10, 11$$

*Memorize MIDPOINT & SLOPE formula

Concept	Formula	Example
SLOPE	$\frac{Y_2 - Y_1}{X_2 - X_1}$	Find the slope of a line that passes through (-5, -3) and (16, -14). $-5, -3$ $16, -14$ x y $\frac{-14+3}{16+5} = \frac{-11}{21}$
MIDPOINT find the middle/ center of segment	$\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2}\right)$	Find the midpoint of a line segment with the given endpoints (-9, 0) and (4, 9) $-9, 0$ $4, 9$ x y $\left(\frac{-9+4}{2}, \frac{0+9}{2}\right) = \left(\frac{-5}{2}, \frac{9}{2}\right)$ or $(-2.5, 4.5)$
DISTANCE Finds length or distance.	$\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$	Find the distance of the line segment between the two points (2, 2) and (7, -3) $2, 2$ $7, -3$ $\sqrt{(7-2)^2 + (-3-2)^2} = \sqrt{50}$ $(5)^2 + (-5)^2$ $25 + 25$ $\sqrt{50}$ 7.1 $5\sqrt{2}$ $25 \cdot 2$
PARTITIONING A SEGMENT ratio a:b	$\left(X_1 + \frac{a}{a+b}(X_2 - X_1), Y_1 + \frac{a}{a+b}(Y_2 - Y_1)\right)$	Given the points A(-3, -6) and B(7, 22), find the coordinates of the P on the line segment AB that partitions AB in a 1:3 ratio. $x_1, y_1 = (-3, -6)$ $x_2, y_2 = (7, 22)$ $a:b = 1:3$ a b $\left(-\frac{1}{2}, 1\right)$ $\left(-3 + \frac{1}{4}(7+3), -6 + \frac{1}{4}(22+6)\right)$ $-\frac{1}{2}$ 1

How do you find the endpoint of a segment given the other endpoint and the midpoint of the segment?

Practice: Find the other endpoint.

① Midpoint: (-3, 3)
Endpoint: (-4, -2)

E_1: -4, -2
E_2: x, y
M: -3, 3

$$\frac{-4+x}{2} = -3$$
$$-4+x = -6$$
$$x = -2$$

$$\frac{-2+y}{2} = 3$$
$$-2+y = 6$$
$$y = 8$$

② Midpoint: (-9, 10)
Endpoint: (-12, 14)

E_1 = -12, 14
E_2 = x, y
M = -9, 10

$$\frac{-12+x}{2} = -9$$
$$-12+x = -18$$
$$x = 6$$

$$\frac{14+y}{2} = 10$$
$$14+y = 20$$
$$y = 6$$

(-6, 6)

Distance, Midpoint, Slope, Partitioning

not on formula sheet

*** Write the midpoint formula**

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Write the distance formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

not on formula sheet

*** Write the slope formula**

$$\frac{y_2-y_1}{x_2-x_1}$$

Partitioning A Segment Formula: $x_1+\frac{a}{a+b}(x_2-x_1), y_1+\frac{a}{a+b}(y_2-y_1)$

1. $A(1,6)$ **and** $B(9,6)$

a) Find the midpoint of $\overline{AB}$

1 6
9 6

$\frac{1+9}{2}, \frac{6+6}{2}$

$(5,6)$

b) find the distance of $\overline{AB}$

$d=\sqrt{(9-1)^2+(6-6)^2}$

$d=\sqrt{64}=8$

c) Find the slope of $\overline{AB}$

$\frac{6-6}{9-1}=\frac{0}{8}$

d) Given points A(4, 10) and B(10, 22), find the coordinates of the point P on directed line segment AB that partitions AB in the ratio 2:1

4, 10
10, 22
2 : 1

$4+\frac{2}{3}(10-4), 10+\frac{2}{3}(22-10)$

$(8,18)$

2. $C(2,-2)$ **and** $D(-6,2)$

a) Find the midpoint of $\overline{CD}$

2, 2
−6, 2
x y

$\frac{2-6}{2}, \frac{-2+2}{2}$

$(-2,0)$

b) find the distance of $\overline{CD}$

$d=\sqrt{(-6-2)^2+(2+2)^2}$

$(-8)^2+(4)^2$

$64+16=\sqrt{80}$

c) Find the slope of $\overline{CD}$

$\frac{2+2}{-6-2}=\frac{4}{-8}=\boxed{-\frac{1}{2}}$

d) Given the points $A(-3,-8)$ **and** $B(9,-14)$**, find the coordinates of the point P on directed line segment** $\overline{AB}$ **that partitions** $\overline{AB}$ **in the ratio 3:2.**

−3, −8
9, −14
3 : 2

$-3+\frac{3}{5}(9+3), -8+\frac{3}{5}(-14+8)$

$\left(\frac{21}{5}, \frac{-58}{5}\right)$

OR

$(4.2, -11.6)$