

Circle Equations 78

standard form: $(x-h)^2 + (y-k)^2 = r^2$

(h, k) is center

$r^2 = \text{radius}$

General form:

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

* must convert to standard form to find center & radius.

Ex. 1: What is the center + radius of the circle:

$$(x-2)^2 + (y+1)^2 = 9$$

Center: $(2, -1)$ * opposite of what's inside
()

radius: $\sqrt{9} = r^2$
 $r = 3$

* Square root r^2 to get radius

Ex 2: Write the equation of a circle with center $(-3, 4)$ + radius 2.

$$(x+3)^2 + (y-4)^2 = 4$$

How to Convert from General to Standard Form. ⁸⁰

* Complete the Square

$$x^2 + y^2 - 2x + 6y - 6 = 0$$

① Rearrange your equation

$$x^2 + x + \underline{\quad} + y^2 + y + \underline{\quad} = \#$$

② circle sign # with x+y

$$x^2 - 2x + 1 + y^2 + 6y + 9 = 6$$

③ take $\frac{1}{2}$ of the x+y, then square it.

$$(x - 1)^2 + (y + 3)^2 = 16$$

④ Set up ()

Center: (1, -3)

$$16 = r^2$$

⑤ Same sign as circled + $\frac{1}{2}$ the #

radius: 4

Writing Equations of Circles

Center:

Radius:

* If given the center + a point on the circle:



Center: ✓

radius: distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

* If given the endpoints of diameter



Center: midpoint formula

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

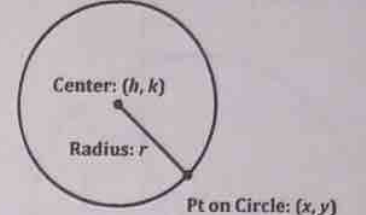
radius: distance formula
from center to an endpoint

Radius When you plug in \rightarrow square it
 When you take out \rightarrow square root it

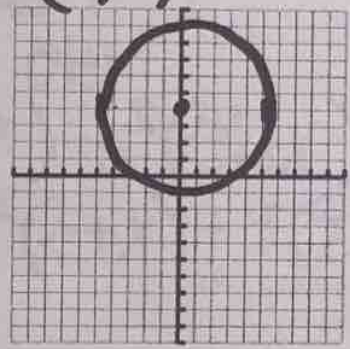
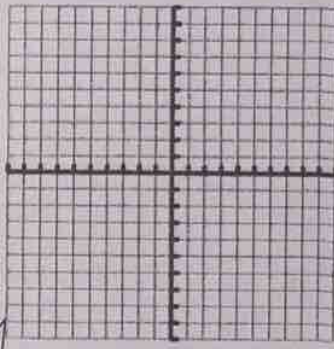
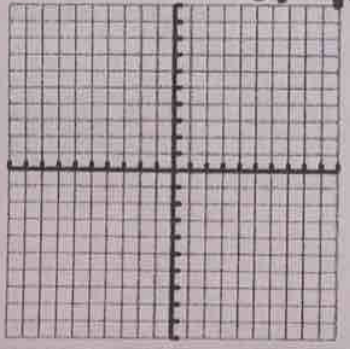
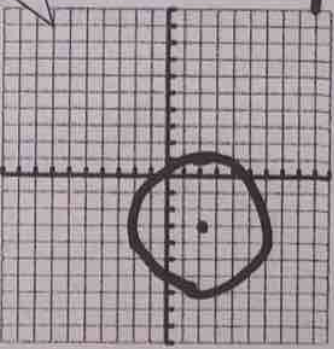
EQUATIONS OF CIRCLES NOTES
 GEOMETRY

NAME: _____
 DATE: _____ PERIOD: _____

- Learning Targets:**
- ✓ Given the equation of a circle find the center and radius
 - ✓ Graph a circle
 - ✓ Given the center and radius write the equation of a circle in standard form
 - ✓ Given the equation of a circle in conic form, complete the square to write it in standard form

<p>Equation of a Circle (Standard Form)</p> $(x - h)^2 + (y - k)^2 = r^2$ <p>where, Center: (h, k) Radius: r</p>	 <p>Center: (h, k) Radius: r Pt on Circle: (x, y)</p>
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For #s 1-4, identify the center of the circle, radius, and sketch the graph.

<p>1) $x^2 + (y - 4)^2 = 25$ $25 = r^2$</p> <p>Center: $(0, 4)$ Radius: 5</p> 	<p>2) $(x - \frac{5}{2})^2 + (y + 3)^2 = 9$ $9 = r^2$</p> <p>Center: $(2.5, -3)$ Radius: 3</p> 
<p>3) $(x + 3)^2 + (y + 1)^2 = 20$</p> <p>Center: $-3, -1$ Radius: $\sqrt{20}$ or 4.5</p> 	<p>4) $(x - 2)^2 + (y + 3)^2 = 16$</p> <p>Center: $2, -3$ Radius: 4</p> 



For #s 5-7, write the equation of the circle in standard form given the following information.

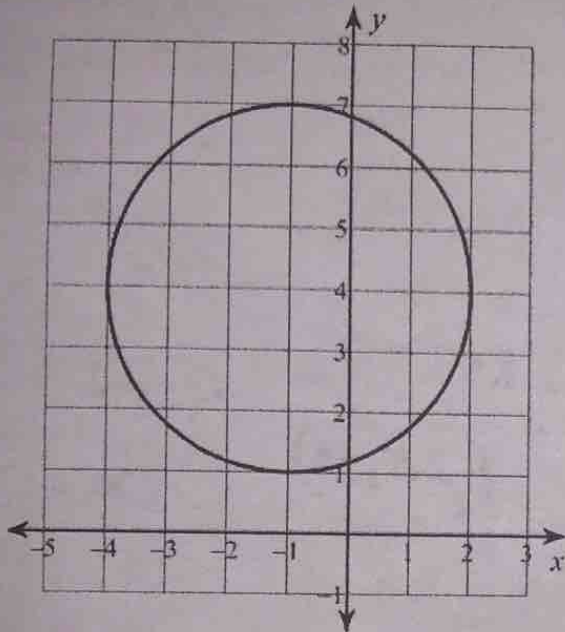
5) Center: $(-15, 12)$, radius = 2

$$(x+15)^2 + (y-12)^2 = 4$$

7) Endpoints of diameter: $(-16, -15)$ and $(-14, 9)$

Center: $\underline{-15, -3}$
 radius: $\underline{\sqrt{145}}$

6)



$$\begin{pmatrix} -16 \\ -14 \end{pmatrix} \begin{pmatrix} -15 \\ 9 \end{pmatrix}$$

$$\frac{-14 + -16}{2}, \frac{9 + -15}{2} =$$

$$d = \sqrt{(-14 + 15)^2 + (9 + 3)^2}$$

$$= \sqrt{1^2 + 12^2}$$

$$= \sqrt{145}$$

$$\begin{pmatrix} -15 \\ -14 \end{pmatrix} \begin{pmatrix} -3 \\ 9 \end{pmatrix}$$

$$(x+15)^2 + (y+3)^2 = \sqrt{145}$$

$$(x+15)^2 + (y+3)^2 = 145$$

Write the equation of the circle in standard form given the following information.

Center: $(6, -8)$, radius = 8

You Try!
#8

9) Prove or disprove that the point $(1, \sqrt{3})$ lies on the circle that is centered at the origin and contains the point $(0, 2)$.

Completing the square with some quadratic equations.

Example: $x^2 - 16x + 60 = 0$

1. Move the constant term to the other side of the equation by adding or subtracting.	1. $x^2 - 16x + 60 - 60 = 0 - 60$
2. Add $(\frac{b}{2})^2$ to both sides.	2. $x^2 - 16x + \underline{64} = -60 + \underline{64}$
3. Factor the left side of the equation into a perfect square trinomial.	3. $(x - 8)^2 = 4$

For #s 10-11, rewrite each equation by completing the square.

10) $x^2 + 10x - 96 = 0$

11) $x^2 - 8x = 56$

- * To find the equation of a circle, you must _____ the square _____!
- * Remember, the standard form of a circle is $(x - h)^2 + (y - k)^2 = r^2$

For #s 12-15, write the equation of the circle in standard form. Then find the center and radius.

<p>12) $x^2 + y^2 - 22x - 12y + 121 = 0$</p> <p>$x^2 - 22x + \underline{121} + y^2 - 12y + \underline{36} = -121$</p> <p>$(x - 11)^2 + (y - 6)^2 = \underline{36}$</p>	<p>Center: <u>(11, 6)</u></p> <p>Radius: <u>6</u></p> <p>$r^2 = 36$ $r = 6$</p>
<p>13) $x^2 + y^2 - 14x - 28y + 220 = 0$</p> <p>$x^2 - 14x + \underline{49} + y^2 - 28y + \underline{196} = -220$</p> <p>$(x - 7)^2 + (y - 14)^2 = \underline{25}$</p>	<p>Center: <u>7, 14</u></p> <p>Radius: <u>5</u></p>

14) $x^2 - 2x + y^2 - 10y = 74$

$$x^2 - 2x + 1 + y^2 - 10y + 25 = 74 + \frac{1}{+25}$$

$$(x-1)^2 + (y-5)^2 = 100$$

Center: $\frac{1, 5}{}$

Radius: $\frac{10}{}$

$$100 = r^2$$

$$10 = r$$

15) $x^2 + y^2 - 28x + 8y + 211 = 0$

$$x^2 - 28x + 196 + y^2 + 8y + 16 = -211 + \frac{196}{+16}$$

$$(x-14)^2 + (y+4)^2 = 1$$

Center: $\frac{14, -4}{}$

Radius: $\frac{1}{}$

$$1 = r^2$$

$$1 = r$$

Write the equation of the circle in standard form by completing the square.

Then, find the radius and center.

$$x^2 + y^2 - 18x + 14y + 129 = 0$$

$$x^2 - 18x + 81 + y^2 + 14y + 49 = -129 + \frac{81}{+49}$$

$$(x-9)^2 + (y+7)^2 = 1$$

$$C: 9, -7$$

$$r = 1$$

You Try!
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